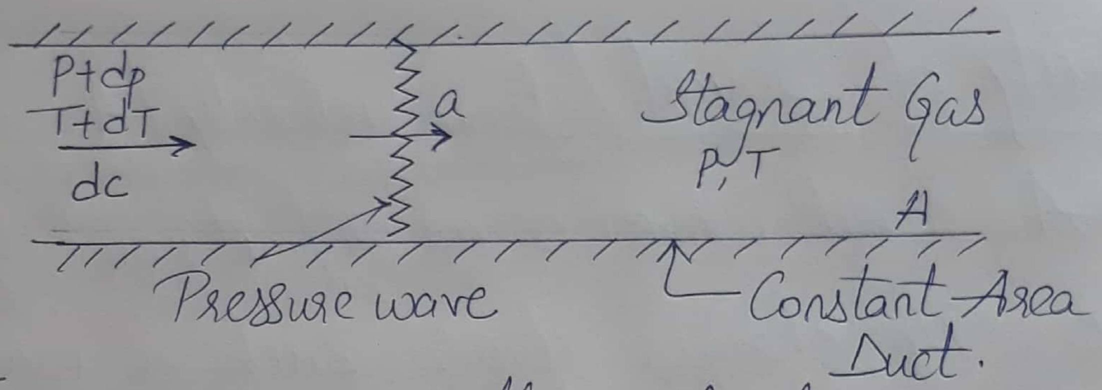
Velocity of Sound / Sonic Velocity / Ac

A sound wave is an infinitesimal pressure wave. The changes across such a wave are small and the speed of process corresponding to these changes is fast. If there is no heat transfer in the system under consideration, changes across the infinitesimal pressure wave can be assumed as rev. adiabatic (or) isentropic.

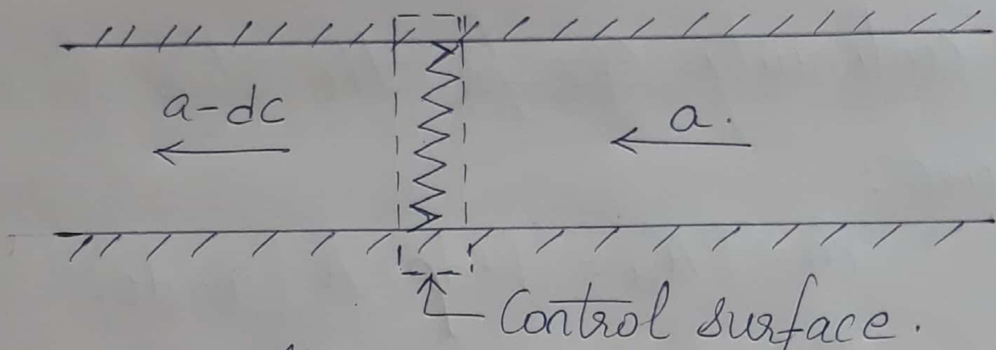
The velocity of sound in a gas depends on its bulk modulus of elasticity (K) or the rate of change of pressure with density ($\partial p / \partial \rho$).

The propagation of an infinitesimal pressure wave into a stagnant gas in a constant area duct as shown in fig.



The wave moves with a velocity a towards the right into the stagnant gas which is at a pressure p and temperature T. The pressure and temp. of the gas that has been traversed by the wave are raised to $(p+dp)$ and $(T+dT)$ and a velocity 'dc' is imparted to the gas. This is the pattern of the process that will be observed by an observer at rest.

When the observer is considered to move with the wave, the stagnant gas at pressure p on the right appears to flow towards the left with a velocity a.



When the flow has passed through the wave to the left its pressure is raised to $(p+dp)$ and the velocity is lowered to $(a-dc)$. Thus the wave can be considered as a stationary wave contained within a control surface through which flow occurs from right to left.

Momentum eq² for this process gives:

$$A[p-(p+dp)] = \dot{m}[(a-dc)-a]$$

$$A\,dp = \dot{m}\,dc \longrightarrow (1)$$

From continuity eq²: $\dot{m} = \rho A a$.

$\therefore$ Eq² (1) changes to $\cancel{A}\,dp = \rho \cancel{A} a\,dc$

$$dp = \rho a\,dc \longrightarrow (2)$$

From continuity eq2 for the two sides of the wave:

$$\dot{m} = \rho \cancel{A} a = (\rho + d\rho)\cancel{A}(a - dc)$$

$$\rho \cancel{a} = \cancel{\rho a} - \rho\, dc + a\, d\rho - d\rho . dc$$

$$\rho\, dc = a\, d\rho \quad \text{[Neglecting } d\rho . dc \text{ term.]} \longrightarrow (3)$$

from eq2 (2) and (3)

$$dp = \rho a\, dc \longrightarrow (2)$$

$$dp = \cancel{\rho} a \left[\frac{a\, d\rho}{\cancel{\rho}}\right] \quad \text{[Sub. 'dc' from eq2 (3)]}$$

$$a^2 = \frac{dp}{d\rho}$$

Since the process has been assumed to be isentropic

$$\boxed{a = \sqrt{\left(\frac{dp}{d\rho}\right)_s}}$$

The above is the fundamental expression for speed of sound in any fluid medium.

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Velocity of Sound in terms of Bulk Modulus (K)

Bulk Modulus of Elasticity, K of a fluid

$$K = -\frac{dp}{dv/v}$$

$\because$ Specific volume is reciprocal of density

$$\frac{dv}{v} = -\frac{d\rho}{\rho}$$

$$\therefore K_s = -\frac{dp}{dv/v} = \frac{dp}{d\rho/\rho}$$

$\therefore$ Velocity of sound in terms of Bulk Modulus.

$$\boxed{a = \sqrt{\left(\frac{dp}{d\rho}\right)_s} = \sqrt{\frac{K_s}{\rho}}}$$

Velocity of sound for isothermal process

For a perfect gas undergoing isothermal process : $\frac{P}{\rho} = C$ (or) $P\rho^{-1} = C$

On differentiating

$$P(-\rho^{-2} d\rho) + \rho^{-1} dp = 0$$

$$\rho^{-1}\left[-\frac{P}{\rho} d\rho + dp\right] = 0$$

$$\frac{dp}{d\rho} = \frac{P}{\rho}$$

For perfect gas: $P = \rho RT$

$$\frac{P}{\rho} = RT$$

$$\therefore \frac{dP}{d\rho} = RT$$

Velocity of sound for isothermal process, $a = \sqrt{\left(\frac{dP}{d\rho}\right)_T}$

$$\boxed{a = \sqrt{RT}}$$

Velocity of sound for adiabatic process

For adiabatic process:

$$\frac{P}{\rho^{\gamma}} = C \quad (or) \quad P\rho^{-\gamma} = C$$

On differentiating

$$P\left(-\gamma \rho^{-\gamma-1} d\rho\right) + \rho^{-\gamma} dp = 0$$

$$\rho^{-\gamma}\left[-\frac{P\gamma}{\rho} d\rho + dp\right] = 0$$

$$\frac{dp}{d\rho} = \frac{P\gamma}{\rho} = \gamma RT \quad \left[\because P = \rho RT\right]$$

$$\boxed{a = \sqrt{\left(\frac{dp}{d\rho}\right)_s} = \sqrt{\gamma RT}} \quad (or) \quad \boxed{a = \sqrt{\frac{P\gamma}{\rho}}}$$

→ velocity of sound for adiabatic/isentropic process.